Loans

Student loans, installment loans (at a retailer) and credit card borrowing will probably be the first loan product students will encounter.

Case Study 1: Zenaida's Student Debt

This case study is about student debt repayment (upon graduation), rather than about taking on student debt at the beginning of college / university. Our premise is that it's more effective to strategize about how much debt one can afford taking when one knows what it takes to repay it.

Case Study 2: Samir's Installment Loan

Installment loans are relevant to young adults. Those are loans many retailers offer at the point of purchase to finance various items, notably electronics, musical instruments, clothes, beauty and wellness products, etc. While marketed as 0% loans, they are of course not free.

With an installment loan, as a buyer you interface with the retailer but behind the scenes are finance companies, often 'fintech' that extend the loan. Here's one example (granted, little financial information is actually available on this webpage): https://paybright.com/en/shop-directory.

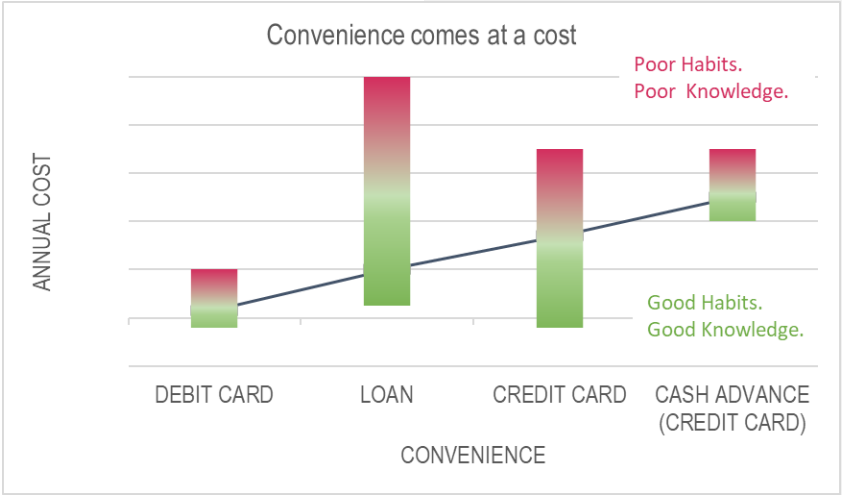
Building on Credit Card Case Study 1 in the earlier chapter (inadvertent borrowing because of a late payment), the student loan and installment loan case studies drive home the message that personal finance problems are not always solved by applying a single mathematical formula. What often works is tabulating monthly cashflows, sometimes for several years. Ability to set this up in Excel (or write some code in a programming language) helps. Paying attention to mundane rules, like proper day count and accounting for holidays, is important.

A framework for thinking about financial products

FinStart thinks of financial products in three dimensions. Convenience is a trade-off for cost (high convenience usually means high cost), while knowledge can mitigate this effect.

On this chart, the height of the bar reflects the range of costs. Loans and credit cards are complicated. Not knowing how they work can increase materially their costs.

Finally, green text in Solutions highlights additional talking points.



Teaching with FinStart

Active learning module:

Coming soon…

How It Works video

Talk Like a Banker (coming soon…)

Take a FinStart Quiz

Reference: Loan pages

- Line of credit / loan
  - Apply
- Student aid
  - Non-repayable
  - Government loans and grants
  - Bank student loans

https://www.finstart.ca/loans-nasml

Case Study 1: Zenaida's Student Loan

Information to use for Problems 1-2

Zenaida graduated from a 5-year university co-op program in May 2020. She has an outstanding Canada Student Loan of $30,000. During her last co-op placement, she received a job offer to start a full-time position starting in September.

Zenaida is planning how to repay her loan. The loan provides for multiple repayment alternatives. She has read the loan agreement and prepared the following summary table.

Feature	Summary
A 6-month non-repayment period	There is a 6-month non-repayment period, between the time you finish school and before you must begin making payments on your Canada Student Loans. Interest will not accrue during the 6-month non-repayment period. Interest may be charged during the non-repayment period on the provincial or territorial portion of some loans. The 6-month non-repayment period starts after you: • Finish your final school term. • Reduce from full-time to part-time studies. • Leave school or take time off school.
Timing of payments	You can make one or more payments at any time after you receive your loan. If you pay off your loan in full before you finish school, you will not pay any interest. After you graduate, you don't have to wait until the 6-month period ends.
Regular Payments	The standard number of payments is 114 if you start paying back 6 months after graduation, or 120 otherwise. Payments you make during the 6-month non-repayment period will be applied directly to the principal of your loan. You may request a longer repayment period of up to 174 months (with the non-repayment period) or 180 months (without it) by changing the terms of your loan agreement.
One-time payments	In addition to regular payments, you can make one-time payments at any time to pay off or reduce your total balance. This is one of the fastest ways to pay off your debt. Payments made while attending school or during the 6-month non-repayment period will be applied directly to the principal of the loan. After the 6-month non-repayment period, you can continue to make one-time payments at any time. One-time payments during your repayment period will be applied to interest first, then to the principal of your loan. You'll still have to make your regular monthly payments.
Interest rate options	You have 2 interest rate options to choose from for your Canada Student Loan: • Floating rate equal to the prime rate. • Fixed rate of prime rate + 2%. Prime rate is calculated using the prime interest rates of the 5 largest Canadian banks, eliminating the highest and the lowest rates and taking the average of the remaining 3.

	Interest rates are subject to change as the prime rate varies. By default, your student loan has a floating rate. You can change to a fixed rate at any time after you start repayment. If you switch to a fixed rate, you will not be able to change back to floating rate.
Fixed interest rate	If you choose fixed rate, your monthly payments will not change throughout the remaining loan repayment period.
Floating interest rate	If you choose floating rate, your total interest payable may fluctuate with changes in the prime rate. Thus, your monthly payments will increase when prime rate rises and decrease when prime rate falls.

Zenaida examined her finances, prepared a budget, and concluded that she should have between $250 and $300 available for monthly loan repayments without sacrificing her other financial objectives. She is going through the alternatives to decide on the best loan repayment option.

Zenaida used "Repayment estimator" posted on Government of Canada page to generate the following eight alternatives.

https://www.canada.ca/en/services/benefits/education/student-aid/grants-loans/repay.html

	Start to repay immediately after finishing school				Start to repay 6 months after finishing school			
	Floating	Fixed	Floating	Fixed	Floating	Fixed	Floating	Fixed
Number of Monthly Repayments	120	120	180	180	114	114	174	174
Prime Rate	2.45%	4.45%	2.45%	4.45%	2.45%	4.45%	2.45%	4.45%
Loan Amount to be Reimbursed	$30,000	$30,000	$30,000	$30,000	$30,000	$30,000	$30,000	$30,000
Monthly Payment Amount	$278.78	$303.55	$196.95	$223.80	$295.24	$323.17	$205.02	$234.29
Total Interest Paid	$3,453	$6,426	$5,450	$10,284	$3,657	$6,841	$5,674	$10,766
Total Amount Paid	$33,453	$36,426	$35,450	$40,284	$33,657	$36,841	$35,674	$40,766

Problem 1 (no calculations required)

Answer the following questions to help Zenaida make the decision. Use Zenaida's research done with the help of Repayment Estimator (previous page).

i) How would the monthly repayment amount change if Zenaida:

a)	Started repayments immediately after finishing school, compared to starting 6 months after graduation?
b)	Increased the number of monthly repayments (i.e. increased the term of the), assuming nothing else changed?
c)	Moved from floating to fixed, assuming nothing else changed?

ii) How would the total interest paid over the life of the loan change if Zenaida:

d)	Started repayments immediately after finishing school, compared to starting 6 months after?
e)	Increased the number of monthly repayments increases, assuming nothing else changed?
f)	Moved from floating to fixed, assuming nothing else changed?

iii) Which loan repayment alternatives would fit within Zenaida budget?

iv) How certain are the estimates provided in the table for the monthly amount of repayment for the floating and fixed rate loan alternatives?

v) Zenaida is considering the floating rate loan with 114 repayments. She knows that prime rate will fluctuate over the next 10-15 years. She is concerned that her monthly payments could change should prime rate increase.

She decided to assess how much it could cost her. Using the same "Repayment Estimator" as before she ran the calculations with different prime rates while keeping all other numbers unchanged. She summarized the results in the table below.

	Start to repay your loan 6 months after finishing school (floating rate)					
Number of monthly repayments	114	114	114	114	114	114
Prime Rate	2.45%	2.95%	3.45%	3.95%	4.45%	4.95%
Loan Amount to be reimbursed	$30,000	$30,000	$30,000	$30,000	$30,000	$30,000
Monthly Payment Amount	$295.24	$302.07	$309.01	$316.04	$323.17	$330.40
Total Interest Paid	$3,657.04	$4,436.46	$5,227.04	$6,028.73	$6,841.49	$7,665.27
Total Amount Paid	$33,657.04	$34,436.46	$35,227.04	$36,028.73	$36,841.49	$37,665.27

g)	How much would her monthly payment increase with each increase of prime rate by 0.50%?
h)	What could Zenaida do to mitigate the impact of prime rate increases on her ability to repay the loan?

Problem 2 (calculations required)

i)	Zenaida decided to make 114 floating rate loan payments starting 6 months after finishing school. She elected that her first payment will be on 15 Dec 2020. When Zenaida was doing her analysis in May 2020, the floating annual interest rate for her loan was 2.45%. She knew that the actual rate for the first month of the loan repayment would be determined on 15 Nov 2020, and it may be different from 2.45%. For now, she expected that her first payment will be $295.24 (as per table in Problem 1). Assuming, that the interest rate remains 2.45% until 15 Jan 2021, calculate the interest amount paid and the outstanding loan amount after the first, the second and the third payment. Which of these three payments results in the smallest repayment of the loan principal?
ii)	Show that the monthly payment of $295.24 per month for 114 months is sufficient to repay Zenaida's $30,000 loan with annual interest rate of 2.45% and monthly compounding. Assume that the payments are at the end of each monthly period.
iii)	Zenaida took a floating rate loan with 114 monthly payments to repay $30,000. She expects monthly payments of $295.24 as the loan has an annual interest rate of 2.45% that compounds monthly. What will be the outstanding balance of the loan after 24 payments if interest rates remained unchanged?
iv)	Zenaida took a floating rate loan with 114 monthly payments to repay $30,000. She expects monthly payments of $295.24 as the loan has an annual interest rate of 2.45% that compounds monthly. After 24 months the rate increases to 3.5%. What will be the new monthly payment Zenaida will have to make?
v)	Zenaida hears that Bank of Canada may further reduce interest rates to help with economic recovery from the Covid-19 recession. She read thi)at the prime rate that is charged on her floating rate loan is driven by how the Bank of Canada sets interest rates. She wonders how much her monthly loan payments would be should prime rate dropped to 1%. Use the ordinary annuity formula to calculate her monthly payment on the $30,000 loan with 114 monthly payments and a 1% annual interest rate, compounded monthly.
vi)	Zenaida thinks that she may be able to make monthly payments of $335 to service her $30,000 floating rate loan at a 2.45% annual interest rate, compounded monthly. However, she is uncomfortable making such a stretched financial commitment. She decides to declare to pay $295.24 for 114 months – the longest loan term, and therefore, the lowest monthly payments – starting 6 months after finishing her studies. Zenaida intends to use the "one-time-payment" feature of the loan and make additional payment of $39.76 each month. Effectively, she would pay $335 each month. a) How many months would it take to repay the loan if prime rate does not change and Zenaida can keep up with $335 monthly payments? b) How much would she save in interest charges by accelerating the repayment when compared to 114 payments of $295.24? c) What do you think about Zenaida loan repayment strategy?

Case Study 2: Samir's Installment Loan (& Credit Card)

Problem 1

It's September 5, 2020. Samir is buying a new laptop for $1,300 after tax. He has not saved the money for the purchase yet and is planning to pay with his credit card that has annual percent rate (APR) of 19.99% for non-cash purchases.

At the checkout Samir is pleasantly surprised to learn that the store is running a back-to-school promotion and offers zero-interest loans. Samir would only pay a processing fee for each payment in the amount of $6.95. The sales rep offered Samir two financing options to choose from:

A. 4 bi-weekly payments of $325.00 net of fees with the first payment today,
B. 12 monthly payments of $108.33 net of fees with the first payment starting in 30 days.

Samir is not sure whether to pay for his purchase with a credit card or apply for one of the merchant's financing alternatives. How would you advise Samir?

Going through the following steps will help you decide:

i. Prepare a payment schedule for alternatives a) and b) using the following table – create separate tables for A and B.

Description	Date	Purchase Amount	Installment Amount Net of Fees	Processing Fee	Installment Amount with Fees
Purchase	05-Sep-20, Sat	$ 1,300.00			
Payment 1					
Payment 2					
Etc.					

ii. Calculate the sum of all interest charges and fees paid for A and B.
iii. For A and B, what percent of the loan principal is the sum of all interest charges and fees? Loan principal is the amount Samir borrows.
iv. What is the term of the loan in A and B, i.e. the number of days for repayment. How does it compare with a credit card?
v. What is the daily rate (of interest and fees combined) for A and B, ignoring compounding?
vi. What is the annual percentage rate (APR) for A and B and how does it compare to the credit card APR Samir sees on his monthly statement? APR ignores compounding and is calculated as the daily rate times the number of days in the year.
vii. Summarize your findings in the following table and provide your recommendation.

	4 Payment Financing	12 Payment Financing	Credit Card Financing
Total interest and fees			
APR			
Loan term			
Other considerations			

Solutions – Loans

Additional 'talking points' are highlighted in green for your convenience.

Case Study 1: Zenaida's Student Loan

Solution 1

i)

a) The repayment amount will be lower if she starts paying her loan immediately after finishing school rather than starting 6 months later. The reason is that the first 6 repayments directly lower the outstanding loan amount as there are no interest charges applied in the 6-month period after completion of studies. Thus, when interest kicks in in the seventh month after graduation, interest is applied to a smaller loan balance.

b) The monthly repayment amount decreases when the number of monthly repayments increases, all other things being equal. Intuitively, you are going to pay a smaller amount if you are going to make more payments.

c) The monthly repayment amount increases when moving from floating to fixed because interest rate charged on a fixed-rate loan is significantly higher than the one on the floating loan, i.e. today 4.45% versus 2.45%.

ii)

d) Total interest paid over the life of the loan is smaller when repayments start immediately after finishing school compared to when they start 6 months later. The reason is that the first 6 repayments directly lower the outstanding loan amount as there are no interest charges applied in the 6-month period after completion of studies. Thus, when interest kicks in in the seventh month after graduation, it is applied to a smaller loan balance, resulting in lower interest paid over the life of the loan.

e) Total interest paid over the life of the loan increases when the number of monthly repayments increases (assuming nothing else changes). The longer the loan is outstanding, the more interest will accumulate.

f) The estimate of the total interest paid over the life of the loan increases when moving from floating to fixed rate loan because interest rate charged on a fixed-rate loan is significantly higher than the one on the floating-rate loan, i.e. today 4.45% versus 2.45%. Interest on the floating rate loan is just an estimate based on the assumption that the current prime rate 2.45% will not change throughout the loan term, i.e. over the next 10-15 years. If prime rate were to significantly increase, it would result in higher actual interest paid over the life of the loan than the current estimate. The amount of interest paid over the life of the fixed rate loan will not be impacted by increases in prime rate. Thus, the actual interest paid over the life of the floating rate loan could exceed that of the fixed rate loan.

iii) Zenaida can consider all repayment alternatives except two: fixed rate loan with the standard number of payments, i.e. 114 and 120.

iv) Actual payments on a floating rate loan can differ significantly from the initial estimate as they change with the changes in prime rate. Actual payments on a fixed rate loan will equal to the estimates as, once fixed, the payments will not change with changes in prime rate. However, there is price to pay for the certainty of a fixed rate loan. The cost is 2% higher than for a floating rate loan.

v)

g) The monthly payment would increase by around $7 for each 0.50% increase in prime rate. Roughly, for each increase in prime of 0.5%, the monthly payment amount will increase by around 2.3%. This is a rough estimate but it illustrates the risk Zenaida is assuming by taking the floating rate loan.

h) To mitigate the impact of increases in prime rate on her ability to repay the loan, Zenaida should take advantage of one-time prepayments that she is allowed to make throughout the life of the loan. It is a very advantageous feature of the loan that the prepayment can be made at any time without any penalty. That feature allows Zenaida to commit herself to the payments that she can make with a high certainty, and at the same time pursue a much more ambitious repayment schedule on the best-effort basis.

Solution 2

i) $29,766.01 will be the outstanding loan balance after Zenaida makes the first payment of $295.24 at the end of the first month.

The monthly loan payment covers interest charges first. The remainder is used to reduce the outstanding loan balance.

In the first month, interest is charged on $30,000, the outstanding loan balance as of the end of the previous month. The annual interest rate at the start of the loan is 2.45%. Thus, the monthly interest rate is 0.204167% = 2.45% / 12. Interest charges are calculated as $30,000 * 2.45% / 12 = $61.25. The outstanding loan balance is $30,000 + $61.25 - $295.24 = $29,766.01

The table below shows how to calculate the interest amount and the outstanding loan balance after each month. Note that monthly interest rate will vary throughout the loan term as it is a floating rate loan.

When interest rate changes, monthly payments are recalculated (using the assumption that the new interest will remain the same to the end of the loan term). Consequently, the monthly interest amounts will be calculated using the new monthly interest rate.

Payment Number	Payment Period End Date	Outstanding Loan Principal	Monthly Interest Rate	Interest Amount	Monthly Loan Payment
	15-Nov-20	$30,000.00			
1	15-Dec-20	$29,766.01 = $30,000 + $61.25 - $295.24	0.204167%	$61.25 = $30,000 *0.204167%	$295.24
2	15-Jan-21	$29,531.54 = $29,766.01 + $60.77 - $295.24	0.204167%	$60.77 = $29,766.01*0.204167%	$295.24
3	15-Feb-21	$29,296.60 = $29,531.54 + $60.77 - $295.24	0.204167%	$60.29 = $29,531.54 *0.204167%	$295.24

ii) One could verify that monthly payments of $295.24 for 114 months are sufficient to repay a $30,000 loan that charges 2.45% compounded monthly by using the step-by-step approach described in the problem above. One could use Excel, Python, Java, VBA etc. to write a small program to perform the calculations. The benefit of such an approach is to generate a full repayment schedule, i.e. the table with 114 rows that shows payment amount, interest amount and the outstanding loan balance for each month.

Alternatively, the formula for ordinary annuity can be used to check that the monthly payments are sufficient. Present Value (PV) of all the payments vendor receives must equal to the value of the loan, namely:

$$PV = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1 + \frac{r}{n}\right)^{-t*n}\right)$$

The Present Value is a finance equivalent of “a bird in hand is worth two in the bush”, i.e. it is better to have something that is certain (the payment today) than having something that is not so certain (the

payment in the future), even if it could possibly be more valuable. The further in the future the payment is scheduled, the less valuable it is, i.e. it has a smaller present value.
The present value of a payment (PMT) scheduled k months from now is less than its stated dollar value, i.e. it is "discounted", as per the formula below:

$$\frac{PMT}{(1 + monthly\ interest\ rate)^k}$$

To justify the discount formula, imagine that you receive the above amount (PV) today. Next, you invest that amount for k months, compounded at a monthly interest rate. After k months, you have the PMT amount.

$$\frac{PMT}{(1 + monthly\ interest\ rate)^k} * (1 + monthly\ interest\ rate)^k = PMT$$

Thus, the value of future PMT amount is discounted at the monthly interest rate.
The monthly interest rate is calculated from the annual interest rate r = 2.45% divided by number of months, i.e. number of compounding periods per year (n = 12). Thus, the monthly interest rate is r/n = 0.204167%. The lender receives Zenaida's monthly payments PMT = $295.26 for t = 9.5 years, i.e. 114 months.

Substituting the numerical inputs to the ordinary annuity formula yields:

$$\text{PV} = \frac{295.24}{\frac{0.0245}{12}} * \left(1 - \left(1 + \frac{0.0245}{12}\right)^{-9.5*12}\right))$$

$$\text{PV} = \frac{295.24}{0.00204167} * (1 - (1 + 0.00204167)^{-114}))$$

PV = 144,607 * 0.207461 = $30,000.31

Rounding of monthly payments to the nearest cent results in a few cents of overpayment. To eliminate the overpayment, the monthly payments should be $295.23716 instead of $295.24, which is not practical.

Below is the derivation of the ordinary annuity formula:

Assume that *i* is a compounding period interest (*i* = r/n in our problem) and *k* is the number of compounding periods (*k* = *t* * *n* in our problem). The sum of all present values of all payments is as follows:

$$PV = \frac{PMT}{(1+i)^1} + \frac{PMT}{(1+i)^2} + \cdots + \frac{PMT}{(1+i)^{k-1}} + \frac{PMT}{(1+i)^k}$$

The above equation is the sum of k terms of a geometric series. The constant ratio between the successive series' terms is 1/(1+i).

Multiplying both sides of the above equation by *(1 + k)* yields the following:

$$PV * (1 + k) = PMT + \frac{PMT}{(1+i)^1} + \cdots + \frac{PMT}{(1+i)^{k-1}}$$

Subtracting the first equation from the second one yields:

$$PV * (1 + k) - PV = \frac{PMT}{(1+i)^k} - PMT$$

$$PV * k = PMT * \left(-1 + \frac{1}{(1+i)^k}\right)$$

Dividing both sides of the above equation by *k* yields the following:

$$PV = \frac{PMT}{k} * \left(1 - \frac{1}{(1+i)^k}\right)$$

Substituting *r*/*n* for *i*, and *t*n* for *k* yields the final version of the equation:

$$PV = \frac{PMT}{\frac{r}{n}} * \left(1 - \frac{1}{\left(1+\frac{r}{n}\right)^{t*n}}\right)$$

$$\text{PV} = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1+\frac{r}{n}\right)^{-t*n}\right)$$

Where *r* is annual interest rate, *n* number of compounding periods per year and *t* is time in years.

iii) After making the 24th payment, the outstanding loan is $24,250.70. This answer is valid only when interest rates remain unchanged at 2.45%

The outstanding amount of the loan is the present value of all remaining payments. Thus, the formula for ordinary annuity can be used:

$$\text{PV} = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1+\frac{r}{n}\right)^{-t*n}\right)$$

To repay the loan, Zenaida will still have 90 payments to make, i.e. t*n = 114 – 24 = 90. The monthly payment PMT = $295.24 and the interest rate is 2.45% compounds monthly. After replacing symbols with numbers we get the following:

$$\text{PV} = \frac{\$295.24}{\frac{2.45\%}{12}} * \left(1 - \left(1+\frac{2.45\%}{12}\right)^{-90}\right)$$

$$\text{PV} = 144{,}607.34693877 * 0.1677002943 = \$24{,}250.70$$

iv) The new monthly payment will be $306.75 after the interest rate is reset to 3.5%. This will be an $11.51 increase in the monthly payment.

From the solution of the previous problem we know that after making the 24th payment the outstanding loan is $24,250.70 if the interest rates remained unchanged at 2.45% for the first 24 months of the loan. As the interest rate is reset to 3.5% (which can happen for a floating rate loan), the payment PMT must be increased so that the loan is repaid in 90 months, i.e. 114 – 24 = 90, to comply with the initial loan term of 114 months.

To solve the problem the ordinary annuity formula

$$\text{PV} = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1+\frac{r}{n}\right)^{-t*n}\right)$$

is re-arranged to show PMT as the dependent variable as follows:

$$\text{PMT} = PV * \frac{r}{n} * \left(1 - \left(1+\frac{r}{n}\right)^{-t*n}\right)^{-1}$$

After replacing symbols with numbers we get the following:

$$\text{PMT} = \$24{,}250.70 * \frac{3.5\%}{12} * \left(1 - \left(1+\frac{3.5\%}{12}\right)^{-90}\right)^{-1}$$

$$\text{PMT} = 70.7312083333 * (0.2305797219)^{-1} = 306.75$$

v) The monthly payments would be $275.97 on the $30,000 loan with 114 monthly payments and the interest rate of 1% compounded annually, i.e. nearly $20 less than when the interest is 2.45%.

To solve the problem the ordinary annuity formula

$$\text{PV} = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1 + \frac{r}{n}\right)^{-t*n}\right)$$

is re-arranged to show PMT as the dependent variable as follows:

$$\text{PMT} = PV * \frac{r}{n} * \left(1 - \left(1 + \frac{r}{n}\right)^{-t*n}\right)^{-1}$$

After replacing symbols with numbers we get the following:

$$\text{PMT} = \$30{,}000 * \frac{1\%}{12} * \left(1 - \left(1 + \frac{1\%}{12}\right)^{-9.5*12}\right)^{-1}$$

$$\text{PMT} = \$30{,}000 * 0.00083333 * (1 - (1 + 0.00083333)^{-114})^{-1}$$

$$\text{PMT} = \$30{,}000 * 0.00083333 * (0.09059074)^{-1}$$

$$\text{PMT} = \$30{,}000 * 0.00919884 = \$275.9652$$

PMT =$275.97 rounded to the nearest cent

vi) Zenaida decides on an aggressive loan repayment plan.

a. It will take 99 months to repay her $30,000 loan if prime rate remains at 2.45% and Zenaida continues to make $335.00 monthly payments.

To solve the problem the ordinary annuity formula

$$\text{PV} = \frac{PMT}{\frac{r}{n}} * \left(1 - \left(1 + \frac{r}{n}\right)^{-t*n}\right)$$

is re-arranged to show the number of compounding periods *t*n* (i.e. the number of months in the current problem) as the dependent variable:

$$\text{t*n} = -\frac{\log\left(1 - \frac{PV}{PMT} * \frac{r}{n}\right)}{\log\left(1 + \frac{r}{n}\right)}$$

or is re-arranged to show number of years *t* as the dependent variable:

$$\text{t} = -\frac{1}{n} \frac{\log\left(1 - \frac{PV}{PMT} * \frac{r}{n}\right)}{\log\left(1 + \frac{r}{n}\right)}$$

After replacing symbols with numbers, we get the following:

$$\text{t*n} = -\frac{\log\left(1 - \frac{\$30{,}000}{\$335} * \frac{2.45\%}{12}\right)}{\log\left(1 + \frac{2.45\%}{12}\right)}$$

$$\text{t*n} = -\frac{\log(0.81716418)}{\log(1.00204167)} = -\frac{-0.08769068}{0.00088578} = 98.998 = 99\ months$$

b. For the accelerated repayment plan, the sum of all payments is 99 * $335 = $33,165. Thus, the total interest amount paid is $33,165 - $30,000 = $3,165.

For the regular repayment plan, the sum of all payments is 114 * $295.24 = $33,657. Thus, the total interest amount paid is $33,657 - $30,000 = $3,657.

The accelerated repayment plan will lower interest costs by $3,657 - $3,165 = $492.

c. Zenaida has a good plan. Her loan repayment strategy takes a full advantage of the no-penalty prepayment feature of student loans. Zenaida intends to follow an aggressive repayment schedule of $335 per month but at the same time she gives herself a buffer… if anything unexpected happens, she can slow down the repayments to the committed amount of $295.24. If she succeeds with her aggressive repayments, she can pay back the loan 15 months earlier and save herself $492. However, as the loan has a floating-rate, the monthly interest rates will fluctuate. Thus, the calculated numbers are only estimates. Nevertheless, when Zenaida follows the aggressive repayment strategy, if the interest rates increase, the impact on her will be smaller. As she repays her loan principal faster, the increased interest will apply to the smaller outstanding loan balance.

Case Study 2: Samir's Installment Loan (& Credit Card)

Solution

1) Payment schedules

A. 4-payment alternative

Description	Date	Purchase Amount	Installment Amount Net of Fees	Processing Fee	Installment Amount with Fees
Purchase and Payment 1	05-Sep-20, Sat	$ 1,300.00	$ 325.00	$ 6.95	$ 331.95
Payment 2	19-Sep-20, Sat		$ 325.00	$ 6.95	$ 331.95
Payment 3	03-Oct-20, Sat		$ 325.00	$ 6.95	$ 331.95
Payment 4	17-Oct-20, Sat		$ 325.00	$ 6.95	$ 331.95

B. 12-payments alternative

Description	Date	Purchase Amount	Installment Amount Net of Fees	Processing Fee	Installment Amount with Fees
Purchase	05-Sep-20, Sat	$ 1,300.00			
Payment 1	05-Oct-20, Mon		$ 108.33	$ 6.95	$ 115.28
Payment 2	05-Nov-20, Thu		$ 108.33	$ 6.95	$ 115.28
Payment 3	05-Dec-20, Sat		$ 108.33	$ 6.95	$ 115.28
Payment 4	05-Jan-21, Tue		$ 108.33	$ 6.95	$ 115.28
Payment 5	05-Feb-21, Fri		$ 108.33	$ 6.95	$ 115.28
Payment 6	05-Mar-21, Fri		$ 108.33	$ 6.95	$ 115.28
Payment 7	05-Apr-21, Mon		$ 108.33	$ 6.95	$ 115.28
Payment 8	05-May-21, Wed		$ 108.33	$ 6.95	$ 115.28
Payment 9	05-Jun-21, Sat		$ 108.33	$ 6.95	$ 115.28
Payment 10	05-Jul-21, Mon		$ 108.33	$ 6.95	$ 115.28
Payment 11	05-Aug-21, Thu		$ 108.33	$ 6.95	$ 115.28
Payment 12	05-Sep-21, Sun		$ 108.33	$ 6.95	$ 115.28

ii. The sum of all interest charges and fees is:
   A. 4-payment alternative: $27.80 (i.e. 4* $6.95) of fees and $0 of interest charges.
   B. 12-payment alternative: $83.40 (i.e. 12 * $6.95) of fees and $0 of interest charges.

iii. As a percent of loan principal:
   A. 4-payment alternative: 2.14% (i.e. $27.80 / $1,300).
   B. 12-payment alternative: 6.42% (i.e. $83.40 / $1,300).

iv. The term for each financing alternatives is as follows:
   A. 4-payment alternative: 42 days (i.e. 3 * 14; the first payment is on purchase date and the last payment is 6 weeks later, i.e. 3 bi-weekly periods).
   B. 12-payment alternative: 365 days (from purchase date to the last payment; the first payment is 1 month after the purchase and the last payment is 12 months after the purchase).

v. The daily rate (of interest and fees combined) for each financing alternative, ignoring compounding:
   A. 4-payment alternative: 0.05092% (i.e. 2.14% / 42).
   B. 12-payment alternative: 0.01758% (i.e. 6.42% / 365).

vi. Annual Percentage Rate (APR) ignores compounding and is calculated as the daily rate times the number of days in the year:
   A. 4-payment alternative: 18.58% (i.e. 365 * 2.14% / 42).
   B. 12-payment alternative: 6.42% (i.e. 365 * 6.42% / 365).
   C. Credit card: 19.99% (as provided in the question - typical rate).

vii. The following table compares the three payment alternatives. The credit card information is provided for reference to stimulate discussion.

Item	4-Payment Financing	12-Payment Financing	Credit Card Financing
Total interest and fees	$27.80	$83.40	It depends: • $0 when full payment is made by the due date • Around $1,364 when only regular minimum monthly payments are made • More when minimum monthly payments are skipped from time to time.
APR	18.58%	6.42%	19.99%
Loan term	365 days	42 days	56 days is the maximum interest-free credit card loan period, on condition that there is no past due balance and the payment is made by the payment due date. 144 months (more than 12 years) if only regular minimum monthly payments are made. Why 56 days? • Minimum 21 days interest-free grace period from the last day of the monthly billing cycle to the payment due date is mandated by regulators. • Some card providers can offer up to 25 days interest-free grace period. • The billing period length varries from 28 to 31 days • When purchase transaction is made on the first day of the monthly credit card billing period the the interest-free grace period can increase up to 25 + 31 days, i.e. 56 days.
Other considerations	-	Penalties for missed payments	-

i. An interest free loan does not mean it is the best deal; various fees as well as penalty interest rates for failing to repay the loan in time must be considered.

ii. APR is not the most reliable measure of loan cost (cheapness).

iii. When choosing a loan one should have a realistic plan to repay it; missing payment dates will result in penalties, retroactive recalculation of interest charges at a steep APR (read a small print) and negative impact on one's credit score.

iv. If Samir needs a year to repay the loan, he should take the 12-payment in-store financing option. At 6.45% APR the loan is low-cost despite a higher dollar amount than the 4-payment alternative.

v. If Samir can repay the loan in 42 days, he can consider his credit card because he can get the financing for 0$ if he satisfies the following conditions:
   - He has no past due balance (i.e. he paid his credit card in full and on time).
   - There are at least 42 days between the purchase date and the payment due date.
   - He will be able to pay in full the balance of his credit card and on time; note that his credit card balance may contain other transactions in addition to the laptop purchase.

vi) If Samir is not sure how his credit card works or does not know his billing period start and end dates, or is not sure he can pay the full balance by the payment date, it will be safer for him to take the 4-payment financing and pay $27.80 for the loan.

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